INFLUENCE OF INTERNAL HEAT SOURCES ON NONSTATIONARY HEAT TRANSFER IN DISPERSE SYSTEMS

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The influence of internal heat sources on the effective thermophysical properties of disperse media is examined and relaxation equations are obtained. It is shown that the heat-conduction equation can be both hyperbolic and elliptic in type.

An extensive literature (see [1-4], say) is devoted to nonstationary heat transfer in disperse systems. Substantial difficulties occur in the determination of the effective heat-conduction coefficients and the different thermal relaxation times in heterogeneous two-phase media. Heat transfer in real disperse systems is often accompanied by heat liberation or absorption because of endo- and exothermal chemical reactions, phase transformations, and a number of other processes that occur in the bulk or on the phase interfaces comprising a heterogeneous medium. Crystallization or solid phase dissolution in a fluid, evaporation, drying, or sublimation in a liquid-drop or powdery cloud, etc., may be examples of such situations.

The presence of internal heat sources is felt in the mean values of the temperature, its gradient and heat flux, which results in both a change in the effective thermophysical characteristics of the disperse medium as compared with the same medium but in the absence of sources, and in certain new physical effects, as is shown in [5], say, in application to stationary heat transfer in disperse systems with phase transformations on the surface of inclusions.

The purpose of this paper is to investigate the influence of thermal sources on nonstationary heat transfer in heterogeneous two-phase media in the simplest example of a medium consisting of a continuous matrix and identical spherical inclusions. It is assumed that there are thermal sources on the surface of the inclusions and in the bulk of the phases, for which the intensity in the temperature range of interest to us can be expressed in the form of the linear function

$$Q_i(T) = L_i^\circ + L_i^1 T. \tag{1}$$

Here L_i°, L_i^1 can have any sign. For instance, the cases $L_i^\circ, L_i^1 < 0$ and $L_i^\circ, L_i^1 > 0$ correspond to endo- and exothermal reactions, respectively, and $L_i^1 < 0 < L_i^\circ$ to heat liberation or absorption during phase transformations, etc.

The investigation is performed on the basis of methods of taking the average over an ensemble of allowable inclusion configurations in combination with self-consistent field theory methods, whose brief exposition can be found in [6, 7], say. For simplicity in the calculations, the case of moderate concentrations is examined (we neglect the requirement of nonoverlappability of the inclusions). It is assumed that the size of the inclusions is much less than all the characteristic linear scales of the macroproblem which, in principle, permits application of continual methods.

In conformity with the theory of taking the average over an ensemble of inclusion configurations [6, 7], the macroscopic heat conduction equation can be written in the form

$$c\frac{\partial\tau(\mathbf{R})}{\partial t} = -\nabla\mathbf{q}(\mathbf{R}) + Q,\ Q = h + m\tau(\mathbf{R}),\ \mathbf{q}(\mathbf{R}) = -\lambda\nabla\tau(\mathbf{R}), \tag{2}$$

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where the following formal relationships hold

$$
\begin{gathered}
\lambda\nabla\tau(\mathbf{R}) = \lambda_0\nabla\tau(\mathbf{R}) + (\lambda_1-\lambda_0)\, n \int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant a} \nabla\tau^*(\mathbf{R},\ \mathbf{R}')\, d\mathbf{R}', \\
Q = n \int\limits_{|\mathbf{R}-\mathbf{R}'|=a} [\overset{\circ}{L}_s + L_s^1\tau^*(\mathbf{R},\ \mathbf{R}')]\, d\mathbf{R}' + \overset{\circ}{L}_0 + \rho(\overset{\circ}{L}_1 - \overset{\circ}{L}_0) + L_0^1\tau(\mathbf{R}) \\
+ (L_1^1 - L_0^1)\, n \int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant a} \tau^*(\mathbf{R},\ \mathbf{R}')\, d\mathbf{R}', \\
c\,\frac{\partial\tau(\mathbf{R})}{\partial t} = c_0\,\frac{\partial\tau(\mathbf{R})}{\partial t} + (c_1-c_0)\, n \int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant a} \frac{\partial\tau^*(\mathbf{R},\ \mathbf{R}')}{\partial t}\, d\mathbf{R}', \\
\nabla\tau^*(\mathbf{R},\ \mathbf{R}') = \frac{\partial\tau^*(\mathbf{R},\ \mathbf{R}')}{\partial\mathbf{R}}.
\end{gathered}
\tag{3}
$$

Here $\tau^*(R,\ R')$ is the temperature at the point R under the condition that the center of an isolated (trial) particle is at the point R' and $|R - R'| \leq a$.

In the general case, the quantities λ, c, and m, with the meaning of an effective heat conductivity, specific heat, and thermal source density, are certain operators and the temperature-independent part of the source density h is a function of the time. In the case of steady nonstationary processes, when initial conditions are not essential in the solution of the macroproblem, it is convenient to apply generalized Fourier transformations in time to (2) and (3), after which these relationships acquire the form

$$
\begin{gathered}
c_\omega i\omega\tau_\omega(\mathbf{R}) = -\nabla\mathbf{q}_\omega(\mathbf{R}) + h_\omega + m_\omega\tau_\omega(\mathbf{R}),\ \mathbf{q}_\omega(\mathbf{R}) = -\lambda_\omega\nabla\tau_\omega(\mathbf{R}), \\
\lambda_\omega\nabla\tau_\omega(\mathbf{R}) = \lambda_0\nabla\tau_\omega(\mathbf{R}) + (\lambda_1-\lambda_0)\, n \int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant a} \nabla\tau_\omega^*(\mathbf{R},\ \mathbf{R}')\, d\mathbf{R}',
\end{gathered}
\tag{4}
$$

$$
\begin{gathered}
h_\omega + m_\omega\tau_\omega(\mathbf{R}) = n \int\limits_{|\mathbf{R}-\mathbf{R}'|=a} [\overset{\circ}{L}_s/i\omega + L_s^1\tau_\omega^*(\mathbf{R},\ \mathbf{R}')]\, d\mathbf{R}' + (\overset{\circ}{L}_1\rho + \\
+ \overset{\circ}{L}_0(1-\rho))/i\omega + L_0^1\,\tau_\omega(\mathbf{R}) + (L_1^1 + L_0^1)\, n \int\limits_{|\mathbf{R}-\mathbf{R}'|\leqslant a} \tau_\omega^*(\mathbf{R},\ \mathbf{R}')\, d\mathbf{R}', \\
c_\omega\tau_\omega(\mathbf{R}) = c_0\tau_\omega(\mathbf{R}) = (c_1-c_0)\, n \int\limits_{|\mathbf{R}'-\mathbf{R}|\leqslant a} \tau_\omega^*(\mathbf{R},\ \mathbf{R}')\, d\mathbf{R}',\ \ i = \sqrt{-1}.
\end{gathered}
\tag{5}
$$

The Fourier transforms of the appropriate operators and functions are marked with the subscript ω in (4) and (5). Unless so stipulated specially below, we omit the subscript ω from the quantities mentioned.

As follows from the general method [6, 7], in order to find the quantities $\nabla\tau^*(R,\ R')$ and $\tau^*(R,\ R')$ it is necessary to solve an auxiliary problem about the perturbations induced by the trial particles in the mean temperature field. The mathematical formulation of this problem has the form

$$
\begin{gathered}
\Delta\tau^+(\mathbf{R}'+\mathbf{r}) - \alpha^2\tau^+(\mathbf{R}'+\mathbf{r}) + f = 0,\ r\geqslant a,\quad \Delta\tau^*(\mathbf{R}'+\mathbf{r}) + \beta^2\tau^*(\mathbf{R}'+\mathbf{r}) + \\
+ \mu = 0,\ r\leqslant a,\quad \tau^+(\mathbf{R}'+\mathbf{r}) = \tau^*(\mathbf{R}'+\mathbf{r}),\ \mathbf{n}\lambda\nabla\tau^+(\mathbf{R}'+\mathbf{r}) + \overset{\circ}{L}_s/i\omega + \\
+ L_s^1\tau^+(\mathbf{R}'+\mathbf{r}) = \mathbf{n}\lambda_1\nabla\tau^*(\mathbf{R}'+\mathbf{r}),\ r = a,\quad \tau^*(\mathbf{R}') \neq \infty,\ \mu = \overset{\circ}{L}_1/i\omega\lambda_1, \\
\alpha^2 = (ci\omega - m)/\lambda, \\
f = h/\lambda,\ \ \beta^2 = (L_1^1 - i\omega c_1)/\lambda_1,\quad \tau^*(\mathbf{R}'+\mathbf{r}) \equiv \tau^*(\mathbf{R},\ \mathbf{R}'),\ \mathbf{r} = \mathbf{R} - \mathbf{R}'.
\end{gathered}
\tag{6}
$$

Here $\tau^+(R' + r)$ has the meaning of a mean temperature at the point R' + r under the condition that the center of a trial particle is at the point R'.

We solve the problem (6) under the additional assumption $|\alpha\ell| \gg 1$, where ℓ is the characteristic dimension of a disperse system. In this case the influence of the system boundary on the temperature near and within the trial particle can be neglected and as follows from [6, 7], the solution of (6) can be written in the form

$$\tau^{+}(\mathrm{R}'+\mathrm{r})=\tau(\mathrm{R}'+\mathrm{r})+\hat{\tau}(\mathrm{R}'+\mathrm{r}),\quad \hat{\tau}(\mathrm{R}'+\mathrm{r})\to 0,\ r\to\infty,$$

where $\hat{\tau}(R' + r)$ is the perturbation induced by the trial particle in the mean temperature field. Consequently, (6) agrees formally with the problem of finding the temperature inside and outside a single particle placed in an infinite medium with the heat conductivity λ, the thermal source density $h + m\tau^{+}(R' + r)$, the specific heat c, and a certain temperature profile $\tau(R' + r)$ at an infinite distance from this particle.

If $|\beta| \lesssim |\alpha|$, then it is convenient to solve (6) analogously to how it is done in [4], by expanding the mean temperature $\tau(R + r)$ near an arbitrary point R in scales of the order a in a Taylor series in the small parameter $\varepsilon \equiv a|\alpha|$. It is here essential to use the assumption that the characteristic scale of the change in $\tau(R)$ is much greater than a.

Expanding $\tau(R' + r)$ near the point R' and introducing a spherical coordinate system with center at the point R', we find $\nabla\tau^{*}(R, R')$ and $\tau^{*}(R, R')$ in the form of series of spherical harmonics (see [4] for more detail about the solution of an analogous problem). The solutions obtained depend here on $\nabla\tau(R')$ and $\tau(R')$ as parameters. Expressing $\nabla\tau(R')$ and $\tau(R')$ in terms of $\nabla\tau(R)$ and $\tau(R)$ by using Taylor series expansions, substituting the results obtained into (5), and integrating, we arrive at a system of transcendental equations in the unknowns λ, c, and m and at equation to determine h. Omitting the intermediate calculations because of their awkwardness, we present the values of the desired quantities λ_ω, c_ω, m_ω, and h_ω, obtained in expanding $\tau(R + r)$ to ε^4 accuracy. Specific calculations were performed for the case $L_1^0 \sim L_1^1\tau^{*}$ (i.e., situations were considered when processes of the type of phase transitions of the first kind, magnetic reversal, depolarization, etc. occur in the bulks of the inclusions):

$$\begin{gathered}
\lambda_\omega=\lambda^\circ+p\lambda^p+i\omega\lambda^\omega+(i\omega\varkappa-p)^{3/2}\lambda',\quad \varkappa=c^\circ a^2/\lambda^\circ,\ c_\omega=c^\circ+pc^p+i\omega c^\omega\\
m_\omega=m^\circ+pm^p+i\omega m^\omega,\quad \lambda^\circ=\lambda_0+(\lambda_1-\lambda_0)\rho 3\lambda^\circ/(\lambda_1+2\lambda^\circ),\ c^\circ=c_0(1-\rho)+c_1\rho,\\
m^\circ=p\lambda^\circ/a^2,\ p=(L_0^1(1-\rho)+L_1^1\rho+3L_s^1/a)\,a^2\lambda^c,\quad \lambda^p=\\
=\left(\frac{p_1}{p}B+3q_s/p\,(\lambda_1/\lambda_0+2)^2+A\right)S,\quad \lambda^\omega=(\varkappa A-\varkappa_1 B)\,S,\\
\lambda'=\frac{3}{2}\,\frac{\lambda^\circ S}{\lambda_1+2\lambda^\circ},\quad S=(\lambda_1-\lambda^\circ)\,\frac{\lambda^\circ(\lambda_1+2\lambda^\circ)}{\lambda_1\lambda_0+2\lambda^{\circ 2}},\quad A=-\frac{2}{3}\,\frac{\lambda^\circ}{2\lambda_1+3\lambda^\circ}-\\
-\frac{1}{5}\,\frac{\lambda^\circ}{\lambda_1+2\lambda^\circ},\quad B=\frac{1}{30}\,\frac{\lambda^\circ(3\lambda_1+2\lambda^\circ)}{(\lambda_1+2\lambda^\circ)^2}+\frac{1}{15},\quad p_1=L_1^1a^2/\lambda_1,\\
\varkappa_1=c_1a^2/\lambda_1,\quad q_s=L_s^1\,a/\lambda^\circ,\\
m^p=\rho\left[-q_s\left(\frac{\lambda_1+5\lambda^\circ}{\lambda_1+2\lambda^\circ}-3\,\frac{\lambda^p}{\lambda^\circ}\right)+\frac{3q_s}{p}\,(q_s+p_1)+\right.\\
\left.+(q_1-q_0)\left(-\frac{3}{10}\,\frac{2\lambda_1+5\lambda^\circ}{\lambda_1+2\lambda^\circ}-\frac{\lambda^p}{\lambda^\circ}+\frac{p_1}{p}\,\frac{2\lambda_1+\lambda^\circ}{6\lambda^\circ}\right)\right]\frac{\lambda^\circ}{a^2},\\
m^\omega=\rho\left[-3q_s\left(\frac{\lambda^\omega}{\lambda^\circ}+\frac{\varkappa}{3}\,\frac{\lambda_1+5\lambda^\circ}{\lambda_1+2\lambda^\circ}\right)-3\varkappa_1q_s\,\frac{\lambda_1}{\lambda^\circ}-q_1\,\frac{\lambda^\omega}{\lambda^\circ}-\right.\\
\left.-(q_1-q_0)\left(\frac{\lambda^\omega}{\lambda^\circ}+\varkappa\,\frac{3}{10}\,\frac{2\lambda_1+5\lambda^\circ}{\lambda_1+2\lambda^\circ}-\varkappa_1\,\frac{2\lambda_1+\lambda^\circ}{6\lambda^\circ}\right)\right]\frac{\lambda^\circ}{a^2},\\
q_i=L_i^1\,a^2/\lambda^\circ,\ i=0,\ 1,\ c^p=-c^\circ\,\frac{\lambda^p}{\lambda^\circ}+(c_1-c_0)\,\rho\left(-\frac{3}{10}\,\frac{2\lambda_1+5\lambda^\circ}{\lambda_1+2\lambda^\circ}+q_s+\frac{p_1}{p}\,\frac{2\lambda_1+\lambda^\circ}{6\lambda^\circ}\right),\\
c^\omega=-c^c\,\frac{\lambda^\omega}{\lambda^\circ}+(c_1-c_0)\,\rho\left(\varkappa_1\,\frac{2\lambda_1+\lambda^\circ}{6\lambda^\circ}-\varkappa\,\frac{3}{10}\,\frac{2\lambda_1+5\lambda^\circ}{\lambda_1+2\lambda^\circ}\right),
\end{gathered}\tag{7}$$

$$h_\omega = h^\circ [1 + h^{\circ\,-1} (L_1^1 - L_0^1 + 3L_s^1/a)(p_s - \overset{\circ}{L}_1 p/3\lambda^\circ p_1) \frac{\rho}{2} -$$

$$- (L_1^1 - L_0^1) \frac{a^2}{\lambda^\circ} \rho \left(-\frac{3}{5} \frac{\lambda^\circ}{\lambda_1 + 2\lambda^\circ} + \frac{1}{2} \right) - 3q_s\rho \left(-\frac{\lambda^\circ}{\lambda_1 + 2\lambda^\circ} + \right.$$

$$\left. + \frac{1}{2} \right) \Big] \frac{1}{i\omega} + \overset{\circ}{L}_1 a^2 \rho \left(L_1^1 - L_0^1 + 3 \frac{L_s^1}{a} \right) \left(1 - \frac{p}{p_1} \right) \frac{\varkappa_1}{6\lambda^\circ (i\omega\varkappa_1 + p_1)},$$

$$h^\circ = \overset{\circ}{L}_0 + \rho(\overset{\circ}{L}_1 - \overset{\circ}{L}_0) + 3L_s\rho/a, \; p_s = \overset{\circ}{L}_s a/\lambda. \tag{7}$$

The quantity λ° is the stationary heat conductivity of the same disperse system but with inert components; it agrees with that obtained in [8]. The quantity λ^p characterizes the influence of the thermal sources on the stationary heat transfer in a disperse medium. The parameter λ^ω governs the directivity and velocity of the relaxation processes in the disperse systems.

Taking account of the equality $q_\omega = -\lambda_\omega \nabla\tau_\omega$, applying the inverse Fourier transform, and neglecting quantities proportional to $p^{3/2}$ and $(\omega\kappa)^{3/2}$, we obtain the relaxation equation

$$\mathbf{q}(\mathbf{R}) = -\left(\lambda^\circ + p\lambda^p + \lambda^\omega \frac{\partial}{\partial t} \right) \nabla\tau(\mathbf{R}), \tag{8}$$

or after evident transformations

$$\left(1 - \frac{\lambda^\omega}{\lambda^\circ} \frac{\partial}{\partial t} \right) \mathbf{q}(\mathbf{R}) = -(\lambda^\circ + p\lambda^p) \nabla\tau(\mathbf{R}). \tag{9}$$

It is seen from (8) and (9) that for $\lambda^\omega > 0$ the gradient of the mean temperature relaxes to the equilibrium value of the heat flux, and for $\lambda^\omega < 0$ the magnitude of the flux relaxes to the equilibrium value of the gradient of the mean temperature.

The third component in the expression for λ_ω in (7) characterizes the influences of the thermal sources on the relaxation processes, where the nature of this influence will be distinctive depending on the relationships between the quantities p and $\kappa\omega$. Let us examine the situation when $|p| > |\kappa\omega|$ and $|p| < |\kappa\omega|$. For $|p| > |\kappa\omega|$ we have $(-p + i\omega\varkappa)^{3/2} \approx (-p)^{3/2} \left(1 - i\omega \frac{3\varkappa}{2p} \right)$. Hence, it is seen in particular that if $p > 0$ (i.e., processes of the exothermal reaction type are predominant in the disperse medium), then such a quasistationary situation is not realized physically. This result is a consequence of the condition $|\alpha| \ell \gg 1$ and is completely analogous to a "thermal explosion" [9]. If $p < 0$, then under evident approximations utilizing the inverse Fourier transform, we obtain

$$\lambda \approx \left[\lambda^\circ + p\lambda^p + |p|^{3/2}\lambda' + \left(\lambda^\omega + \frac{3}{2} |p|^{1/2} \varkappa\lambda' \right) \frac{\partial}{\partial t} \right]. \tag{10}$$

It is seen from (10) that the presence of heat sources increases or decreases the relaxation time for $\lambda^\omega\lambda' > 0$ and $\lambda^\omega\lambda' < 0$, respectively.

For $|p| < \omega\kappa$ we have $(-p + i\omega\varkappa)^{3/2} \approx (i\omega\varkappa)^{3/2} (1 - 3p/2\varkappa i\omega)$. Limiting ourselves to the terms written down for the expansion, we obtain after applying the inverse Fourier transforms

$$\lambda \approx \lambda^\circ + p\lambda^p + \lambda^\omega \frac{\partial}{\partial t} + \lambda' \sqrt{\varkappa/\pi} \left(\varkappa \int_{-\infty}^{t} \frac{\partial t'}{\sqrt{t - t'}} \frac{\partial^2}{\partial t'^2} - \frac{3}{2} p \int_{-\infty}^{t} \frac{\partial t'}{\sqrt{t - t'}} \frac{\partial}{\partial t'} \right) W(t, t'), \; W(t, t')\varphi(t) = \varphi(t'). \tag{11}$$

In this case the relaxation equation becomes nonlocal in time. The first integral in (11) is analogous to that obtained in [4] for the inert phase case, and the second characterizes the influence of the thermal sources on memory processes.

After the inverse Fourier transforms, as is easily seen from (7), the effective values of the specific heat c as well as the parameters m and h acquire the form

$$c = c^\circ + c^p p + c^\omega \frac{\partial}{\partial t}, \quad m = m^\circ + pm^p + m^\omega \frac{\partial}{\partial t}, \tag{12}$$

$$h = h^\circ \left[1 + h^{\circ -1} (L_1^1 - L_0^1 + 3L_s^1/a)(p_s - L_1^\circ p/3\lambda^\circ p_1) \frac{\rho}{2} - \frac{L_1^1 - L_0^1}{\lambda^\circ} a^2 \rho \left(\frac{3}{5} \frac{\lambda^\circ}{\lambda_1 + 2\lambda^\circ} + \frac{1}{2} \right) - 3q_s \rho \left(\frac{\lambda^\circ}{\lambda_1 + 2\lambda^\circ} + \frac{1}{2} \right) \right].$$

Within the limits of the accuracy chosen, the heat-conduction equation can be obtained in the following form after evident transformations

$$\lambda^\circ \Delta \tau (\mathbf{R}) + h - h^\circ p \frac{\lambda^p}{\lambda^\circ} + m^\circ \left[1 + p \left(\frac{m^p}{m^\circ} - \frac{\lambda^p}{\lambda^\circ} \right) \right] \tau (\mathbf{R}) +$$

$$+ \left[p \left(c^\circ \frac{\lambda^p}{\lambda^\circ} - c^p - \frac{\lambda^\omega}{\lambda^\circ} \right) + m^\omega - c^\circ \right] \frac{\partial \tau (\mathbf{R})}{\partial t} - c^\circ \left(\frac{c^\omega}{c^\circ} - \frac{\lambda^\omega}{\lambda^\circ} \right) \frac{\partial^2 \tau (\mathbf{R})}{\partial t^2} = 0. \tag{13}$$

It hence follows that the heat-conduction equation can be elliptic for $c^\omega/c^\circ < \lambda^\omega/\lambda^\circ$, hyperbolic for $c^\omega/c^\circ > \lambda^\omega/\lambda^\circ$, and not just hyperbolic as is asserted in [2, 3], say, from thermodynamic or phenomenological considerations. An equation analogous to (13) has been obtained in [4] for the temperature, which is average just in the continuous phase, for the case of inert comopnents of the disperse medium.

It is seen from (13) that the thermal sources influence the inertial properties of disperse media, which is characteristic for precisely heterogeneous systems.

Let us emphasize that expressions (7)-(13) are obtained for steady nonstationary processes. In case when the initial conditions of the acroproblem are essential, it is pertinent to use the Laplace rather than the Fourier transform in (2) and (3). In particular, it should here be expected that terms dependent functionally on the time would appear in the expressions for h and the relaxation relationships.

NOTATION

a, radius of the inclusions; c, specific heat; λ, heat conductivity; L_j^i, coefficients introduced in (1); ℓ, dimension of the disperse medium; n and ρ, numerical and bulk concentrations of the inclusions, respectively; n, unit vector normal to the particle surface; h and m, coefficients introduced in (2); T and τ, true and mean temperature; τ^* and τ^+, temperature inside and outside the trial particle; $\hat{\tau}$, perturbation induced by the trial particle in the mean temperature; α, β, μ, f, parameters introduced in (6); ε, a dimensionless parameter characterizing the ratio between the inclusion radius and the characteristic scale of variation of τ; ω, Fourier transform parameter λ^j, c^j, m^j ($j = 0, p, \omega$), λ', p, p_1, p_s, h°, κ, κ_1, S, A, B, q_i ($i = s, 0, 1$) introduced in (7); W(t, t'), an operator introduced in (11); t, time; $\phi(t)$, an arbitrary function of the time; Q, thermal source density; q, heat flux density. Superscripts: 0, 1, continuous and discrete phases; s, thermal sources on the inclusion surface; quantities without superscripts refer to the effective homogeneous medium.

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HEAT TRANSFER AND CRYODEPOSIT PROPERTIES IN SOLID-STATE CONDENSATION

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An expression is derived for the effective thermal conductivity, which agrees with experiment. A study is made of heat transfer in solid-state condensation, and working formulas are derived.

Solid-state condensation or desublimation has not been extensively discussed in the literature, in contrast say to boiling. Solid-state condensation is used in some processes in chemical engineering and in cryogenic pumps. If the temperature of the cold surface is below the triple point, the vapor condenses directly to the solid state. It is familiar that water vapor will deposit as frost from air on cold surfaces. In all cases, the deposit adversely affects the heat transfer as it insulates the surface, and the removal requires the process or pump to be shut down.

The growth of the solid phase is a complicated nonstationary process involving various heat- and mass-transfer mechanisms. It can be represented as a boundary-value problem with a mobile boundary, which is characterized by heat and mass transfer through the growing layer of solid phase. The processes within the layer are such that the boundary layer can be considered as quasistationary.

The layer of solid phase is a porous body. In accordance with the conditions of formation, it has low density and high porosity, or conversely high density and low porosity. The temperature gradient in the layer produces vapor migration in condensation. Therefore, the density and thermal conductivity alter during the condensation.

Solid-state condensation on cold surfaces has been examined with an apparatus containing a vacuum chamber, in which a thermally insulated vessel containing liquid nitrogen was placed, which had an open condensation surface of diameter 56 mm. Condensable vapors were admitted (freon 13 and acetone) through a system including an RS-3A rotameter, damping capillary, fine-adjustment leak, and nozzle with porous baffle providing a vapor flow uniform over the cross section. The heat flux was determined from the loss of liquid nitrogen by boiling. The temperature of the cold surface was monitored by four copper–constantan thermocouples and recorded by an F-30 digital voltmeter. The pressure was measured with an oil gauge read by means of a cathetometer, which provided a sensitivity of 0.01 mm in the range 0.1-1.5 mm Hg. The surface temperature and the layer thickness were determined with a mobile probe fitted with a micrometer screw giving a displacement accuracy of 0.01 mm. The maximum layer thickness was 5 mm.

The deposit was observed and photographed with an illumination system. As there are no standard data, we determined the following parameters of the monolithic solid in separate experiments: heat sublimation, density, and thermal conductivity.

The formation of the solid phase begins with the production of thin needles growing from the cold surface. Then the tops of the needles, which had attained a length of 2-3 mm, began to produce whiskers, which were interwoven. The structure became more complicated. The growth of the solid continued on the outside. There were considerable effects on the structure

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